

Due Friday, February 28, 2025.

Write your homework *neatly, in pencil*, on blank white $8\frac{1}{2} \times 11$ printer paper. Always *write the problem*, or at least enough of it so that your work is readable. If the problem involves a function, write the function. If the problem involves an equation, write the equation. Use words, and when appropriate, *write in sentences*.

Definition 1. Define the *natural logarithm* to be the function

$$\log : (0, \infty) \rightarrow \mathbb{R} \quad \text{given by} \quad \log(x) = \int_1^x \frac{1}{t} dt.$$

We have shown that \log is bijective.

Definition 2. Define the *natural exponential function* to be the inverse of the natural logarithm. Thus

$$\exp : \mathbb{R} \rightarrow (0, \infty) \quad \text{such that} \quad \exp(x) = y \Leftrightarrow x = \log(y).$$

Define the number e by

$$e = \exp(1).$$

For $a \in (0, \infty) \setminus \{1\}$ and $x \in \mathbb{R}$, define

$$a^x = \exp(x \log(a)) \quad \text{so that} \quad e^x = \exp(x).$$

We have shown that $\frac{d}{dx} \exp(x) = \exp(x)$, so $\int \exp(x) dx = \exp(x) + C$.

Problem 1 (Thomas §7.3 # 9). Solve for y where

$$\ln(y - 1) - \ln(2) = x + \ln x.$$

Problem 2 (Thomas §7.3 # 21). Find $\frac{dy}{dx}$ where

$$y = xe^x - e^x.$$

Problem 3 (Thomas §7.3 # 35). Find $\frac{dy}{dx}$ where

$$y = \int_0^{\ln x} \sin(e^t) dt.$$

Problem 4 (Thomas §7.3 # 47). Evaluate the definite integral

$$\int_{\ln 4}^{\ln 9} e^{x/2} dx.$$

Problem 5 (Thomas §7.3 # 53). Evaluate the indefinite integral

$$\int \frac{e^{1/x}}{x^2} dx.$$

Problem 6 (Thomas §3.5 # 36). Find $\frac{dr}{d\theta}$ where

$$r = \sec \sqrt{\theta} \tan \left(\frac{1}{\theta} \right).$$

Problem 7 (Thomas §3.8 # 9). Let

$$f(x) = \sqrt[3]{x}.$$

Find the linearization of f at $x = 8$, and use it to estimate $\sqrt[3]{8.5}$.

Problem 8 (Thomas §7.3 # 67). Let

$$f(x) = e^x - 2x.$$

Find the absolute maximum and minimum values of f on $[0, 1]$.

Problem 9. Let R be the region bounded by $x = 0$, $x = \ln(5)$, $y = 0$, and $y = e^x$. Find the volume of the solid obtained by revolving R about the x -axis

Problem 10. Let R be the region bounded by $x = 0$, $x = \ln(5)$, $y = 0$, and $y = e^x$. Find the volume of the solid obtained by revolving R about the y -axis